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ABSTRACT

The fundamental opposition of two types of discourse, mathematical proof and argumentation to the problem of validation in mathematics is explored. The naturalistic study of interaction in classrooms suggests the possibility of a mathematical argumentation to which students have access by the practice of discussions ruled by socio-mathematical norms that would emerge from the interactions between teacher and student. In such an approach, the construction of a mathematical rationality and argumentation are tightly connected. While there may be no such thing as "mathematical argumentation," there is argumentation in mathematics. It is suggested that argumentation constitutes an epistemological obstacle to the learning of mathematical proof, and more generally the learning of proof in mathematics. (SLD)

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Is argumentation an obstacle ? Invitation to a debate...

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The first diagnosis of the possible sources of difficulty in the teaching and learning of mathematical proof to be proposed was the nature of the most natural didactical contract emerging from the positions of the student and the teacher with respect to the knowledge in question. Since the teacher is the guarantor of the legitimacy and epistemological validity of what is being constructed, it follows that the student would be deprived of an authentic access to a *problématique* of truth and proof. Surmounting this difficulty, which is inherent to the nature of didactical systems, can be looked for in situations permitting the devolution to the students of mathematical responsibility for what they produce. Such a devolution implies the disappearance of the teacher in the process of decision-making in the course of solving a problem, to be replaced by an effort by the students to construct autonomous methods of proof.

Argumentation,
a *problématique* resulting from the study of social interactions

Social interaction among the students has appeared clearly as one of the powerful levers for encouraging the process of devolution to the students of mathematical responsibility for their activity and their productions. To the point that certain people have come to regard social interaction as the most excellent response to the problems posed. The rhetoric of those who maintain such a position is based essentially on the idea that relegating the teacher to the role of guide or organizer of learning will make an opening, simply by this movement of retreat, for an authentic construction of knowledge.

I studied such situations, as did other researchers, in the course of the eighties. The works of this period seem to me to have confirmed the productive and essential character of social interaction, but they also, and perhaps above all, revealed that by its very nature this type of interaction creates social processes and behaviors which run counter to the construction of a mathematical, and more generally scientific, *problématique* of proof by

the students. These processes and behaviors can be assembled under a single reference theme, that of argumentation. I cited at the time, in support of the didactical conjecture according to which for the students a *problématique* of argumentation was in opposition to a mathematical *problématique* of proof, the theses resulting from the work of Perelman, notably:

> "Whereas mathematical proof in its most perfect form is a series of structures and of forms whose progression cannot be challenged, argumentation has a non-constraining character. It leaves to the author hesitation, doubt, freedom of choice; even when it proposes rational solutions, non is guaranteed to carry the day." (Perelman 1970 p.41)

Even if we don't go as far as a conception of mathematical proof in its most perfect form, which we do in taking the point of view of the practice of mathematicians, there remains a fundamental opposition in the terrain of the contribution of these two types of discourse to a *problématique* of validation. This opposition, as is frequently forgotten, affects not only the question of proof, but that of refutation. For example, the ad hoc treatment of counterexamples, as noted in several different research experiments, suggests that counterexamples are seen as objections rather than as refutations indicating a contradiction.

Argumentation,
a *problématique* resulting from the study of verbal productions

The relationship between argumentation and mathematical proof has been an object of past study from a linguistic and cognitive perspective. There, it is a matter of exploring the cognitive complexity of each type, the relationship to knowledge that it involves or favors, basing the study on the analysis of the text and the uses of language. To situate the *problématique* of such approaches, taking for my own use a formulation of Jean-Blaise Grize: arguing is doubtless a finalized activity, but it is a discursive activity(discourse being understood, whatever its subject, to be a social activity.)

Argumentation and mathematical proof are distinguished less by the types of the corresponding texts - Raymond Duval emphasizes that the discursive distance between them is tiny - than by the status and functioning of the statements, and thus in the end that of the knowledge in play. Argumentation, since its functioning appears to emerge naturally from the common practices of discourse, would not permit the identification of the modification of the status and of the functioning of the knowledge required by mathematical work, and in return the modification of the discourse itself.

The examination of the relationship of argumentation and mathematical proofing this approach, which is centered on the analysis of discourse, seems to me to support the conjecture of a conflicting relationship between the two genres when viewed from the perspective of the learning of mathematics. Raymond Duval concluded of it that"the development of argumentation, even in its most elaborated forms, does not open the way to mathematical proof. A specific and independent learning process is necessary where deductive reasoning is concerned" (here using, in my opinion in a faulty way, deductive

reasoning as synonymous with mathematical proof). His conclusion is that mathematical proof involves "specific and independent" learning.

Nonetheless, the naturalistic study of interactions in the class, as conducted by, for example, Paul Cobb and his group, suggests the possibility of a mathematical argumentation to which the students have access by the practice of discussions ruled by sociomathematical norms which would emerge from the interactions between teacher and student (the teacher being regarded as a representative of the mathematical community.) In such an approach, construction of a mathematical rationality (mathematical proof is not actually taught as such) and argumentation are tightly connected.

Different theoretical conceptions of argumentation

The diversity which we may observe among the *problématique*s of argumentation and of its relations with mathematics, notably with mathematical proof, is in my view basically due to profound differences in the theoretical research in this domain. I will not make an analysis here of the different *problématique*s of argumentation, but making use of the synthesis proposed by Christian Pantin in his Essais sur l'argumentation [Essays on argumentation], I shall attempt to give an idea of the importance of taking this diversity into account. Three authors, by the contrast of their *problématique*s and their distance, can be used to provide a system of benchmarks by reference to which one can situate works on argumentation: Chaïm Perelman, Stephen Toulmin and Oswald Ducrot.

Following Perelman, one considers argument to be characterized less by the mastering of its object than of the listener. It is finalized less by the establishment of the validity of a statement than by its capacity to convince a listener. Taking up the formulation of Pantin, a statement in this conception has a value of reason, or even truth, from the moment that an individual accepts it.

Toulmin, on the other hand, relates the validity of a statement first to that of the structure (the rationality) of the discourse which defends it and thus makes this validity depend fundamentally on the premises within a community (of a domain) of reference from the moment that this community agrees to these rules.

> "[An argument] is the exposition of a controversial thesis, the examination of its consequences, the exchange of proofs and of good reasons supporting it and a well- or ill- established closure."

Independently of the domains, argumentative discourse is organized in a compound way which permits passage from the givens to a conclusion under the generally implicit control of a "license to infer" (this schema can be augmented by indicators of strength or restriction making it possible to take into account a possible uncertainty about the inference.)

Ducrot places argumentation at the heart of the activity of discourse. As Pantin emphasizes, in this *problématique* "one cannot not argue." The structure of the sequence of arguments plays a determining role: the strength of an argument comes neither from "natural" characteristics nor from rational characteristics, but from its location in the

statement. It is by the structure that one signifies, that one shows an orientation which makes it possible to receive "R as the intentional goal of P", or "R as possibly following from P." The analysis of conjunctions (connecting words) has a particular importance for Ducrot because it is they which make the information contained in a text subject to its global argumentative intention. The polyphony of the connectors, in the end, makes it possible to represent in the discourse not only the speaker but also his potential protagonist. "P but Q" suggests a subject holding with P to which the speakerobjects Q.

We note that the reference to one or another of these conceptions ofargumentation is likely to make us adopt a different position with regard to what the argumentcan represent in the practice of mathematics, notably with the intent of teaching andin relationship with mathematical proof. Following Toulmin, it seems possible toenvisage a solultion of continuity from argumentation to mathematical proof, and why notconsider mathematical proof as a particular argumentative genre (I do not know whetherToulmin marked the opposition between argumentation and mathematical proof that theother approaches emphasize.) On the other hand, the existence of such a solutionappears doubful if one works from within the framework proposed by Perelman andDucrot.

The risks of recognizing a "mathematical argumentation"

My position at this point in my reflections leads me to consider that inargumentation there is a double activity of persuasion and validation. Though one may doubt it incertain disputes where good faith is not a requirement, one can, on the other hand,makeit a hypothesis in the scientific perspective excluding trickery and lying (the idealposition without which our objective loses all sense.)

- Argumentation seeks to convince a listener, but does that mean that with Perelman we reduce it to doing just that?
- Argumentation introduces an object, the validity of a statement. But the sources of argumentative competence are in natural language and in practices whose rules are frequently of a profoundly different nature from those required by mathematics, and carry a profound mark of the speakers and circumstances. In that measure, I would like to say that the theoretical frameworks of Toulmin and Ducrot, in a fashion nonetheless less radical than Perelman, still give a central place to social interactions and relations (though Ducrot might well protest that point.) Now, if we postulate the sincerity of the speakers in the field of scientific practice, argumentation must satisfy the conditions for entry into a *problématique* of knowledge which involves the decontextualization of the discourse, the disappearance of the actor and of the duration. All conditions which run counter to the profound nature of argumentation whatever the *problématique* that one wishes to associate with it.

I would maintain thus that there is no mathematical argumentation in thefrequently suggested sense of an argumentative practice in mathematics which ischaracterized by the

fact that it escapes certain of the constraints present for mathematical proof.This does not mean that all discourse in mathematics aimed at establishing the validity of astatement has always had and can always have the characteristics of a proof. It is a richness oflatin languages which permits us to make a distinction between "preuve" and"démonstration" [which, lacking that richness, we have translated respectively as "proof" and "mathematical proof"! (V.W.)] imposing on the former the requirements linkedto the construction of an oeuvre of knowledge, without submission to therequirementsof form of the latter.

If there is no such thing as mathematical argumentation, there doesnonetheless exist argumentation in mathematics. The resolution of problems, in which I wouldlike to say that there are no holds barred, is the context in which to develop theargumentative practices using means which could be used elsewhere (metaphor, analogy,abduction, induction, etc.) but which disappear in the construction of a discourseacceptablewith regard to the rules specific to mathematics. I will give a capsule description ofthe place I think possible for argumentation in mathematics, using the sense of the conceptof cognitive unity of theorems coined by our Italian colleagues:

> Argumentation is to a conjecture what mathematical proof is to a theorem.

A consequence that some may consider catastrophic is that, as it is the case for problem-solving, argumentation flatly refuses any attempt to be taught directly(I am not, of course, confusing the teaching of argumentation with the teaching of rhetoric.)

Argumentation,
epistemological obstacle to the teaching of mathematical proof

At the conclusion of this short essay, from the perspective of teaching andlearning, I arrive at supporting neither the thesis of continuity nor that of a rupturebetween argumentation and mathematical proof (or proof in mathematics), but atproposing the recognition of the existence of a relationship which is complex and is part ofthemeaning of both: argumentation constitutes an epistemological obstacle to the learningof mathematical proof, and more generally of proof in mathematics.

To understand mathematical proof is first to construct a particular relationshipwith knowledge as the goal of a theoretical construction, and then to give up thefreedom which one could give oneself as a person in the play of an argument. Becausethis movement towards mathematical rationality can only be accomplished bybecoming effectively aware of the nature of validation in this discipline, it will provokethedouble construction of argumentation and of mathematical proof. Argumentation incommon practice is spontaneous, as is emphasized by those who study discourse. Forgedin familiar exchanges, in the playground, in multiple and frequently insignificant circumstances, a student's argumentative competence is in the image of familiarpractices: it goes on its own. Mathematics class is one of the few places where theexistence of that practice can be revealed because it suddenly appears inadequate (but situationsfor creating this awareness are difficult to construct). In my eyes it would even bean error of epistemological character to let students believe, by a sort of Jourdain effect,that they are

capable of producing a mathematical proof when all they have done is argue.

Finally, and this has to do with a point which I have not approached but whichmust not be forgotten, the major point separating argumentation and mathematical proof is the necessity of the latter to exist relative to an explicit axiom system. Possibly because of bad memories left by the New Math, the idea of linking mathematical proof and axiom systems seems often to provoke anxiety, if not downright opposition, and yet: in the long run, can one do otherwise without reducing mathematical proof to a particular rhetoric or mathematics to a language game?

Some Readings

Ducrot O. (1980) Les échelles argumentatives. Paris : les éditions de Minuit.

Duval R. (1991) Structure du raisonnement déductif et apprentissage de la démonstration. Educational Studies in Mathematics 22(3) 233-261.

Duval R. (1992) Argumenter, démontrer, expliquer : continuité ou rupturecognitive ? Petit X 31, 37-61.

Perelman Ch. (1970) Le champ de l'argumentation. Bruxelles : Presses Universitaires.

Plantin C. (1990) Essais sur l'argumentation. Paris : Editions Kimé.

Yackel E, Cobb P. (1996) Sociomathematical norms, argumentation, andautonomy in mathematics. Journal for Research in Mathematics Education 27(4) 458-477.